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Nonlinear Regulation of Space Station: A Geometric Approach

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This paper presents a new approach to control system design for the nonlinear regulation and angular momenta management of the space station in the presence of disturbance torque based on geometric control theory. The aerodynamic disturbance torque is treated as the output of an exosystem that is Poisson stable. An output zeroing submanifold for the pitch, yaw, and roll dynamics is obtained. A control law is obtained such that in the closed-loop system the trajectories converge to this manifold and the desired equilibrium state is attained. For the synthesis of the controller, the states associated with the exosystem are generated using measurement on the attitude angles and angular rates. Simulation results are presented to show that in the closed-loop system, attitude regulation and momenta management are accomplished in spite of the presence of the aerodynamic disturbance inputs.

I. Introduction

ATTITUDE control of space vehicles employing control moment gyros (CMGs) is an interesting problem. The equations of motion of the space station are described by nonlinear differential equations. Often, attitude control system design using linear control theory[1-9] is obtained. However, linear control systems are designed based on the assumption that the perturbation in attitude angles are small. For large changes in orientation of space vehicles employing momentum exchange devices, nonlinear controllers have been described in the literature.[10-14] An adaptive control design has been presented in Ref. 15. In recent papers[1,3-9] interesting approaches to CMG momentum management and attitude control of the space station using linear quadratic optimization, pole assignment techniques, and game theory have been reported. However, these control system designs are based on linearized models of the space station. An exact feedback linearization technique has been used in Ref. 16 to derive an attitude control system. This control law has a singularity at 45-deg pitch angle. Input-output feedback linearization has been used in Ref. 17 to design a controller for the space station. However, the effect of disturbance torque has not been treated in Refs. 16 and 17.

We present in this paper a new approach to attitude control system design of the space station employing control moment gyros. For simplicity, here CMGs are considered as ideal torquers; however, the CMG gimbal dynamics should be included in the further development of control systems. A geometric approach[18,19] to control system design is taken for the nonlinear pitch, yaw, and roll axis regulation of the space station and for the momentum management in the presence of aerodynamic disturbance torque inputs. The unknown disturbance torques contain sinusoidal functions of the orbital frequency and twice the orbital frequency, besides constant terms, and can be considered to be generated by a dynamic system called an exosystem. This exosystem is Poisson stable in the neighborhood of the origin. The output variables chosen for regulation are the roll CMG momentum and the pitch and yaw angles. An output zeroing submanifold (a hypersurface) is obtained by solving an associated partial differential equation in the closed form. A control law is derived such that the output zeroing manifold is attractive. The system trajectories are thus attracted to this manifold. On this manifold the roll CMG momentum and the pitch and yaw angles are constant, whereas the space station rocks about the roll axis in spite of continuously acting aerodynamic torque. Since the

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state variables of the exosystem are unknown, an observer is designed for estimating these states. These estimated states are used for the synthesis of the controller.

The organization of the paper is a follows. The space station dynamics and control problem are described in Sec. II. Section III presents the attitude control law based on the geometric approach. The observer design is presented in Sec. IV and Sec. V presents the results of digital simulation.

II. Mathematical Model and Control Problem

Consider the space station in a circular orbit. An orbital frame of reference with its origin at the center of mass of the space station is chosen. The axis of the reference frame are chosen such that the roll axis is in the flight direction, the pitch axis is perpendicular to the orbital plane, and the yaw axis points toward the Earth. The orientation of the space station with respect to the reference frame is obtained by a pitch-yaw-roll (θ_2-θ_3-θ_1) sequence of rotations, where θ_1, θ_2, and θ_3 are the roll, pitch, and yaw angles. The nonlinear equations of motion have been derived in Ref. 1 and can be written as follows.

Space station dynamics:

$$\begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} \dot\omega_1 \\ \dot\omega_2 \\ \dot\omega_3 \end{bmatrix} = - \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} + 3n^2 \begin{bmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} + \begin{bmatrix} -u_1 + w_{1d} \\ -u_2 + w_{2d} \\ -u_3 + w_{3d} \end{bmatrix} \tag{1}$$

where

$$c_1 \triangleq -\sin\theta_2 \cos\theta_3$$

$$c_2 \triangleq \cos\theta_1 \sin\theta_2 \sin\theta_3 + \sin\theta_1 \cos\theta_2$$

$$c_3 \triangleq -\sin\theta_1 \sin\theta_2 \sin\theta_3 + \cos\theta_1 \cos\theta_2$$

Attitude kinematics:

$$\begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_2 \end{bmatrix} = \frac{1}{\cos\theta_3} \begin{bmatrix} \cos\theta_3 & -\cos\theta_1 \sin\theta_3 & \sin\theta_1 \sin\theta_3 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 \cos\theta_3 & \cos\theta_1 \cos\theta_3 \end{bmatrix}$$

$$\times \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} + \begin{bmatrix} 0 \\ n \\ 0 \end{bmatrix} \tag{2}$$

CMG momentum:

$$\begin{bmatrix} \dot h_1 \\ \dot h_2 \\ \dot h_3 \end{bmatrix} + \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{3}$$

where the orbital angular velocity is $n = 0.0011$ rad/s, (ω_1, ω_2, ω_3) are the body-axis components of absolute angular velocity, (I_{11}, I_{22}, I_{33}) are the moments of inertia, I_{ij} ($i \neq j$) are the products of inertia, (h_1, h_2, h_3) are the body-axis components of CMG momentum, (u_1, u_2, u_3) are the body-axis components of control torque, and (w_{1d}, w_{2d}, w_{3d}) are the body-axis components of aerodynamic torque. The n^2-dependent term is the gravitational torque.

We shall treat the question of control of the space station for the configuration described in Ref. 1, which requires a large pitch. The complete equations of motion for this configuration have been derived in Ref. 1. These are

$$I_1\ddot\theta_1 + (1 + 3\cos^2\theta_2)n^2(I_2 - I_3)\theta_1 - n(I_1 - I_2 + I_3)\dot\theta_3$$

$$+ 3(I_2 - I_3)n^2(\sin\theta_2 \cos\theta_2)\theta_3 = -u_1 + w_{1d}$$

$$I_2\ddot\theta_2 + 3n^2(I_1 - I_3)\sin\theta_2 \cos\theta_2 = -u_2 + w_{2d}$$

$$I_3\ddot\theta_3 + (1 + 3\sin^2\theta_2)n^2(I_2 - I_1)\theta_3 + n(I_1 - I_2 + I_3)\dot\theta_1 \tag{4}$$

$$+ 3(I_2 - I_1)n^2(\sin\theta_2 \cos\theta_2)\theta_1 = -u_3 + w_{3d}$$

$$\dot h_1 - nh_3 = u_1 \qquad \dot h_2 = u_2 \qquad \dot h_3 + nh_1 = u_3$$

Equations (4) are derived from Eqs. (1-3) assuming that θ_2 is large but that the roll and yaw attitude errors are small. Similarly to Ref. 1 it is assumed here that the products of inertia are small, and these are neglected. We have retained nonlinear functions of the pitch angle in the model. Here $I_i \triangleq I_{ii}$, $i = 1, 2, 3$. We observe from Eqs. (4) that the pitch axis dynamics are decoupled from the roll and yaw dynamics. Such uncoupling of pitch axis motion simplifies the attitude control problem.

The model of the disturbance torque w_{id}, $i = 1, 2, 3$, has been provided in Ref. 1 and is given by

$$w_{id} = A_{1i} + A_{2i}\sin(nt + \phi_{1i}) + A_{3i}\sin(2n + \phi_{2i}) \tag{5}$$

where the magnitudes A_{1i}, A_{2i}, A_{3i} and the phases ϕ_{1i} and ϕ_{2i} are assumed unknown for control design. The cyclic component of the orbital rate is caused by the Earth's diurnal bulge, whereas the cyclic torque at twice the orbital rate is caused by the rotating solar panels. It is pointed out that the approach of this paper can be easily extended to the case when the aerodynamic torque model requires additional sinusoidal functions of higher frequencies. The disturbance torque w_{id} can be treated as output of an exosystem given by ($i = 1, 2, 3$)

$$\begin{aligned} \dot w_{i0} &= 0 \\ \dot w_{i1} &= -nw_{i2} \\ \dot w_{i2} &= nw_{i1} \\ \dot w_{i3} &= -2nw_{i4} \\ \dot w_{i4} &= 2nw_{i3} \\ \dot w_{id} &= w_{i0} + w_{i1} + w_{i3} \end{aligned} \tag{6}$$

We note that any w_{id} of the form of Eq. (5) can be obtained by the proper selection of the initial conditions of the exosystem (6). Define $w_i = (w_{i0}, \bar w_i^T)^T \in R^5$, $w_0 = (w_{10}, w_{20}, w_{30})^T$ and $\bar w_i = (w_{i1}, w_{i2}, w_{i3}, w_{i4})^T \in R^4$ (T denotes transposition). Then the exosystem can be written as

$$\begin{pmatrix} \dot w_{i0} \\ \dot{\bar w}_i \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \Lambda \end{pmatrix} \begin{pmatrix} \bar w_{i0} \\ \bar w_i \end{pmatrix}$$

$$\triangleq A_d w_i \tag{7}$$

where 0 denotes appropriate matrices and

$$\Lambda = \begin{bmatrix} 0 & -n & 0 & 0 \\ n & 0 & 0 & 0 \\ 0 & 0 & 0 & -2n \\ 0 & 0 & 2n & 0 \end{bmatrix}, \qquad A_d = \begin{pmatrix} 0 & 0 \\ 0 & \Lambda \end{pmatrix} \tag{8}$$

Define the state vector of the exosystem $w = (w_1^T, w_2^T, w_3^T)^T \in R^{15}$, the state vector of the space station $x = (\theta_1, \dot{\theta}_1, h_1, \theta_2, \dot{\theta}_2, h_2, \theta_3, \dot{\theta}_3, h_3)^T \in R^9$, $S = \mathrm{diag}\,(A_d, A_d, A_d)$, and $P_i = [b_i\; b_i\; 0\; b_i\; 0]$, where $b_i = 1/I_i$.

Then the system (4) including the exosystem can be written in a state variable form as

$$\dot{x} = \begin{bmatrix} \dot{\theta}_1 \\ f_2(x) \\ nh_3 \\ \dot{\theta}_2 \\ f_5(x) \\ 0 \\ \dot{\theta}_3 \\ f_8(x) \\ -nh_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -b_1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -b_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -b_3 \\ 0 & 0 & 1 \end{bmatrix} u + \begin{bmatrix} 0 & 0 & 0 \\ P_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & P_3 \\ 0 & 0 & 0 \end{bmatrix} w$$

$$\triangleq f(x) + Bu + Pw \tag{9}$$

$$\dot{w} = Sw$$

where $f(x)$, B, and the 9×15 matrix P are defined in Eq. (9), $u = (u_1, u_1, u_3)^T$ and

$$f_2(x) = [-k_1(\theta_2)\theta_1 + k_2\dot{\theta}_3 - k_4(\theta_2)\theta_3]/I_1$$

$$f_5(x) = [-3n^2(I_1 - I_3)\sin\theta_2 \cos\theta_2]/I_2$$

$$f_8(x) = [-k_5(\theta_2)\theta_3 - k_2\dot{\theta}_1 - k_3(\theta_2)\theta_1]/I_3$$

$$k_1(\theta_2) = (1 + 3\cos^2\theta_2)n^2(I_2 - I_3)$$

$$k_2 = n(I_1 - I_2 + I_3)$$

$$k_3(\theta_2) = 3(I_2 - I_1)n^2 \sin\theta_2 \cos\theta_2$$

$$k_4(\theta_2) = 3(I_2 - I_3)n^2 \sin\theta_2 \cos\theta_2$$

$$k_5(\theta_2) = (1 + 3\sin^2\theta_2)n^2(I_2 - I_1)$$

The function k_i depends on θ_2.

We are interested in designing a control system such that the attitude angles and the CMG momenta of the space station can be controlled using CMGs in spite of the disturbance torque. For the control system design, it is essential to make an appropriate choice of the controlled variables $y(t)$ such that the closed-loop system has desirable behavior. Although one would like to obtain a fixed orientation of the space station with respect to the orbital frame it turns out that this is not feasible with bounded CMG momenta. Divergence of h_1 and h_3 occurs since the frequency of the sinusoidal components of the control inputs u_1 and u_3 required for canceling w_{1d} and w_{3d} coincides with the natural frequency n of the dynamics of h_1 and h_3. Instead, motivated by the results in Ref. 1, we select roll CMG momentum and pitch and yaw angles as the controlled output variables, that is,

$$y(t) = (h_1\; \theta_2\; \theta_3)^T \tag{10}$$

Suppose it is desired to regulate y to $y^* = [0, \theta_2^*(w_0), \theta_3^*(w_0)]^T$. It will be seen later that the choice of equilibrium attitude angles $\theta_2^*(w_0)$ and $\theta_3^*(w_0)$ as functions of the constant component w_0 of the disturbance input results in a closed-loop system in which the CMG momenta are bounded. We observe that the choice of y^* gives constant target pitch and yaw angles, and the target roll CMG momentum is zero. The computation of θ_2^* and θ_3^* will be presented later.

Now we associate with the system (9) an output error vector defined by

$$e(t) = y - y^* = [h_1, \theta_2 - \theta_2^*(w_0), \theta_3 - \theta_3^*(w_0)]^T \tag{11}$$

We are interested in deriving a feedback control law $u(x, w)$ that is a function of x and w such that the equilibrium state $x = 0$ of the closed-loop system is asymptotically stable when $w = 0$, and the output $e(t)$ of Eqs. (9) and (11) asymptotically tends to zero for the initial conditions $[x(0), w(0)] \in V \subset X \times W$, a neighborhood of the origin, where $x \in X$ and $w \in W$.

III. Nonlinear Regulation

In this section based on a geometric approach, a control law for nonlinear regulation will be derived. The resolution of this problem requires derivation of an output zeroing submanifold M (a hypersurface) of the state space, namely, the graph of a mapping $x = \pi(w)$, $\pi = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8, \pi_9)^T \in R^9$ and a control input $u = \beta(w)$. The submanifold $x = \pi(w)$ is rendered invariant by the control input $u = \beta(w)$. Invariance of M implies that if the initial state lies on this manifold $\{x(0) = \pi[w(0)]\}$, then $x(t)$ remains on M for $t > 0$. The manifold M is such that when the trajectory evolves on M, the corresponding output error e is zero. That is, the pitch and yaw angles are constant, and the roll CMG momentum h_2 is zero. Based on a result in Ref. 18, we state the following theorem.

Theorem 1: There exists a control law $u(x,w)$ for the system (9) such that the nominal closed-loop system ($w = 0$) is asymptotically stable in the first approximation and for $[x(0), w(0)] \in V \subset X \times W$, a neighborhood of (0,0), the error e tends to zero as $t \to \infty$, if and only if there exists a smooth mapping $x = \pi(w)$, with $\pi(0) = 0$, and an input $u = \beta(w)$, with $\beta(0) = 0$, both defined in a neighborhood of $W^0 \subset W$ of 0, satisfying conditions

$$(\partial\pi/\partial w)Sw = f[\pi(w)] + B\beta(w) + Pw \tag{12}$$

$$[\pi_3(w), \pi_4(w) - \theta_2^*(w), \pi_7(w) - \theta_3^*(w)]^T = 0 \tag{13}$$

The condition (12) expresses the fact that the manifold M defined by $x = \pi(w)$ in the state space of the system is rendered invariant by means of the feedback law $u = \beta(w)$. From Eq. (13) it follows that the error vector $e = y - y^*$ [given in Eq. (11)] is zero at each point of this manifold. For this reason M is called an output zeroing submanifold M of the system (9). Since $u = \beta(w)$ on M, in view of the differential equations associated with h in Eq. (4), it follows that $\beta(w)$ must not include any constant functions, otherwise the CMG momenta will grow without bound.

The derivation of the control law requires the solution of the partial differential equation (12) indicated in Theorem 1 and the determination of the function $\beta(w)$.

Theorem 2: Consider the partial differential (12) and Eq. (13) of Theorem 1. There exist smooth mappings $x = \pi(w)$ [with $\pi(0) = 0$] and $u = \beta(w)$ [with $\beta(0) = 0$] both defined in the neighborhood $W^0 \subset W$ of 0, which solve Eqs. (12) and (13) and are given by $\{\pi(w) = [\pi_1(w), \ldots, \pi_9(w)]^T\}$

$$\pi(w) = \begin{bmatrix} \theta_1^*(w_0) + c^T\bar{w}_1 + d^T\bar{w}_3 \\ c^T\Lambda\bar{w}_1 + d^T\Lambda\bar{w}_3 \\ 0 \\ \theta_2^*(w_0) \\ 0 \\ (w_{22} + 0.5w_{24})/n \\ \theta_3^*(w_0) \\ 0 \\ -(a^T\bar{w}_1 + b^T\bar{w}_3)/n \end{bmatrix} \tag{14}$$

$$\beta(w) = \begin{bmatrix} a^T\bar{w}_1 + b^T\bar{w}_3 \\ w_{21} + w_{23} \\ -(a^T\Lambda\bar{w}_1 + b^T\Lambda\bar{w}_3)/n \end{bmatrix} \tag{15}$$

where a, b, c, and d are appropriate 4×1 real vectors [Eqs. (36) and (37)].

Proof: For a proof see the Appendix.

The target pitch and yaw angles are given in Eqs. (25) and (29) in the Appendix. According to Eq. (14), whenever $x \in M$, pitch and yaw angles take constant values $\theta_2^*(w_0)$, and $\theta_3^*(w_0)$ and $h_1 = 0$. However, roll angle θ_1 and CMG momenta h_2 and h_3 are periodic functions of time since π_1, π_6, and π_9 linearly depend on the sinusoidal functions $\bar{w}_i$.

It has been indicated earlier and also described in Ref. 1 that for the linearized dynamics of the space station in the presence of disturbance inputs w_{id} it is not possible to accomplish attitude regulation to some fixed orientation θ^* with bounded CMG momenta, where $\theta = (\theta_1, \theta_2, \theta_3)^T$ and $\theta^* = (\theta_1^*, \theta_2^*, \theta_3^*)^T$. This can also be proved analytically for the nonlinear system (9). In this case, the output error vector is $e = (\theta - \theta^*)$; and for the existence of control law for zeroing the error, one must solve for $\pi(w)$ and $\beta(w)$ satisfying Eq. (12) and $[\pi_1(w) - \theta_1^*, \pi_4(w) - \theta_2^*, \pi_7(w) - \theta_3^*] = 0$ instead of Eq. (13) to obtain the zero-error manifold M. Boundedness of h requires that $\beta(w)$ should be free of constant functions. When one attempts to solve for π and β, one obtains an inconsistent set of equations. (The details are not given here.)

Now that the output zeroing submanifold M given by $x = \pi(w)$ has been obtained, we proceed to derive the control law for regulation. For regulation, one must design a controller such that in the closed-loop system, the trajectories beginning from any initial condition $x(0)$ converge to M. The linear approximation of the system (9) when $w = 0$ plays an important role in the solution of the regulation problem. For $w = 0$, the linearized system representing Eq. (9) about $x = 0$ is

$$\dot{x} = Ax + Bu, \qquad A = \frac{\partial f(0)}{\partial x} \tag{16}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 & 0 & 0 & a_{27} & a_{28} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & n \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{54} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ a_{81} & a_{82} & 0 & 0 & 0 & 0 & a_{87} & 0 & 0 \\ 0 & 0 & -n & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where $a_{21} = -b_1k_1(0)$, $a_{27} = -b_1k_4(0)$, $a_{28} = b_1k_2$, $a_{54} = -3n^2(I_1 - I_3)b_2$, $a_{81} = -b_3k_3(0)$, $a_{82} = -b_3k_2$, $a_{87} = -b_3k_5(0)$, and B is defined in Eq. (9). For the space station, the matrix pair (A, B) is stabilizable, and there exists a feedback matrix F such that the matrix $(A + BF)$ is Hurwitz. Here the gain matrix F has been obtained using the linearized model at $x = 0$. In general, for large values of θ_i^*, one has to determine F based on a linearized model about $\theta_i = \hat{\theta}_i^*$, an estimate of θ_i^*.

Following Refs. 17 and 18, the control law for error regulation is given by

$$u(t) = \beta(w) + F[x - \pi(w)] \tag{17}$$

where $\beta(w)$ and $\pi(w)$ are given in Theorem 2. In the closed-loop system, the error vector $e(t) = [h_1(t), \theta_2(t) - \theta_2^*(w_{20}), \theta_3(t) - \theta_3^*(w_0)]^T \to 0$ as $t \to \infty$, and the closed-loop system is asymptotically stable when $w = 0$.

The control law (17) is a function of the trajectory error $x - \pi(w)$ and causes the convergence of the trajectory $x(t)$ to the manifold M when the initial state lies outside M. Thus as $x(t) \to \pi(w)$, in view of Eq. (14), $h_1(t) \to 0$ and θ_2 and θ_3 tend to θ_2^* and θ_3^*, respectively. However, oscillatory responses for θ_1, h_2, and h_3 persist. The asymptotic trajectories of these variables are $\pi_1(w)$, $\pi_6(w)$, and $\pi_9(w)$, and the control inputs converge to the sinusoidal functions $\beta(w)$ derived in Theorem 2.

IV. Observer Design

For the synthesis of the control law (17), it is essential to know the state of the exosystem (6). But these are not measurable. Thus, for the implementation of the controller, one needs to construct an estimate $\hat{w}$ of the vector w. We assume that the state vector x is measurable and proceed to design an observer.

Since roll, pitch, and yaw dynamics have independent disturbance inputs and θ and $\dot{\theta}$ are measurable, one can design decoupled observers to estimate w_1, w_2, and w_3. For the design of the observer we consider the system (i = 1, 2, 3)

$$\frac{d}{dt}\begin{bmatrix} \theta_i \\ \dot{\theta}_i \\ w_{i0} \\ w_{i1} \\ w_{i2} \\ w_{i3} \\ w_{i4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_i & b_i & 0 & b_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -n & 0 & 0 \\ 0 & 0 & 0 & n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2n \\ 0 & 0 & 0 & 0 & 0 & 2n & 0 \end{bmatrix} \begin{bmatrix} \theta_i \\ \dot{\theta}_i \\ w_{i0} \\ w_{i1} \\ w_{i2} \\ w_{i3} \\ w_{i4} \end{bmatrix} + \begin{bmatrix} 0 \\ g_i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{18}$$

$$\triangleq A_{io}x_{io} + g_{oi}(x, u_i)$$

where $x_{io} = (\theta_i \;\; \dot{\theta}_i \;\; w_{i0} \;\; w_{i1} \;\; w_{i2} \;\; w_{i3} \;\; w_{i4})^T$, and $(g_1, g_2, g_3) = [f_2(x) - b_1u_1, f_5(x) - b_2u_2, f_8(x) - b_3u_3]$. The observation equation is

$$y_{io} = C_o x_{io} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x_{io}$$

The observer is chosen of the form

$$\dot{\hat{x}}_{io} = A_{io}\hat{x}_{io} + L_i(y_i - C_o\hat{x}_{io}) + g_{oi}(x, u_i) \tag{19}$$

Let the estimation error be $\tilde{x}_{io} = (x_{io} - \hat{x}_{io})$. Then the state estimation error satisfies

$$\dot{\tilde{x}}_{io} = (A_{io} - L_iC_o)\tilde{x}_{io} \tag{20}$$

The matrix L_i is chosen such that $(A_{io} - L_iC_o)$ is a Hurwitz matrix and thus $\tilde{x}_{io} \to 0$, as $t \to \infty$. In the control law (17), the estimate $\hat{w}$ is substituted in place of w giving

$$u(t) = \beta(\hat{w}) + F[x - \pi(\hat{w})] \tag{21}$$

The estimates $k_i[\theta_2^*(\hat{w}_{20})]$ in place of $k_i[\theta_2^*(w_{20})]$ are used in Eq. (A16) for the computation of the parameters a, b, c, and d. These parameters are used in Eq. (14) to obtain π and β. This completes the control system design.

V. Simulation Results

In this section the results of digital simulation are presented. The system parameters and the disturbance inputs are given in the Appendix. The matrix F is chosen such that the eigenvalues of $(A + BF)$ are: pitch dynamics $[-1.6n, (-1.5 \pm j1.5)n]$ and roll and yaw dynamics $[-0.33n, -1.6n, (-1.5 \pm j1.5)n, (-2.1347 \pm 3.0591j)n]$.

The matrices L_i are chosen such that the eigenvalues of the observers are: roll dynamics $[-0.68n, (-0.66 \pm 1.51j)n, (-1.5 \pm 0.84j)n, (-1.02 \pm 0.29j)n]$, yaw dynamics $[-0.23n, (-0.66 \pm 1.51j)n, (-1.5 \pm 0.84j)n, (-1.02 \pm 0.29j)n]$, and pitch dynamics $[-1.5n, (-1.05 \pm 0.68j)n, (-1.04 \pm 0.72j)n, (-1.05 \pm 0.68j)n]$.

Simulation is done using nominal and off-nominal space station parameters for large angle perturbations in the pitch angle. The initial conditions chosen are (θ_i, deg; $\dot{\theta}_i$, deg/s; and h_i, ft-lb-s)

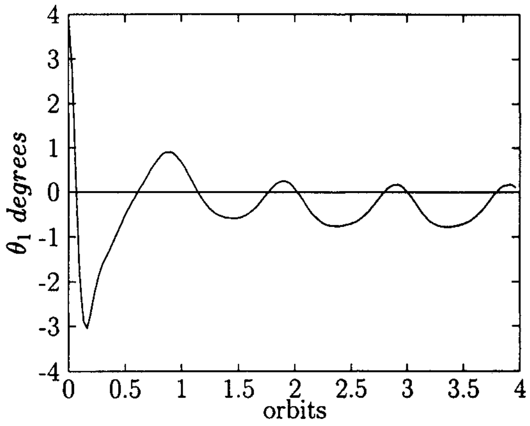

Fig. 1a Attitude regulation of nominal parameters: θ_1 roll angle.

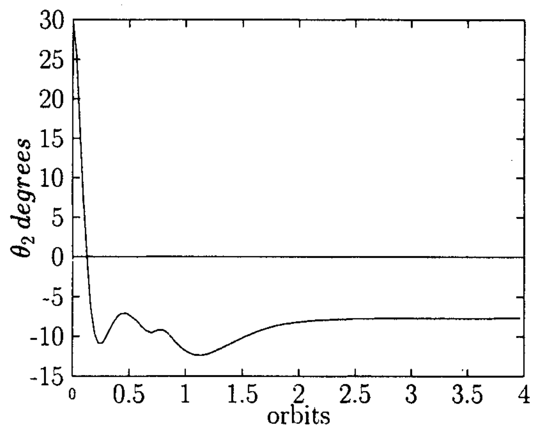

Fig. 1b Attitude regulation of nominal parameters: θ_2 pitch angle.

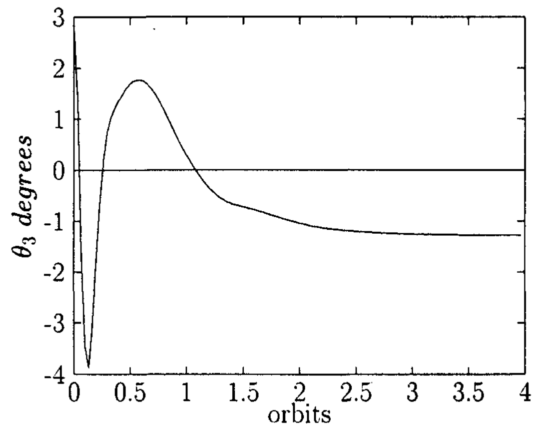

Fig. 1c Attitude regulation of nominal parameters: θ_3 yaw angle.

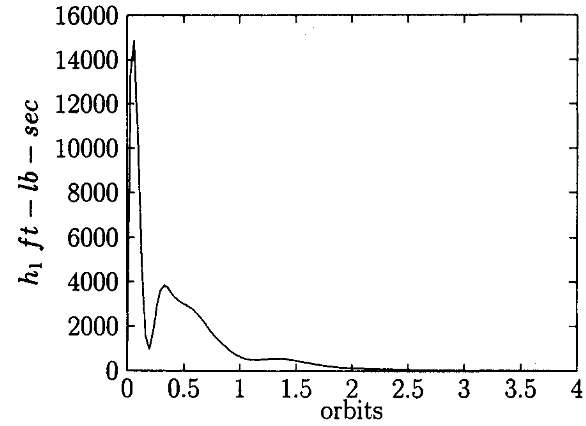

Fig. 1d Attitude regulation of nominal parameters: h_1 momentum.

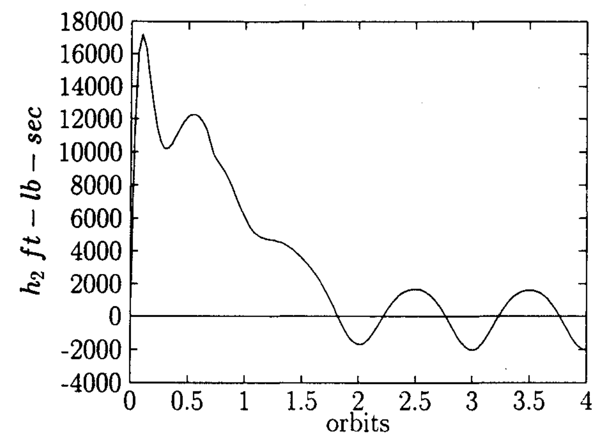

Fig. 1e Attitude regulation of nominal parameters: h_2 momentum.

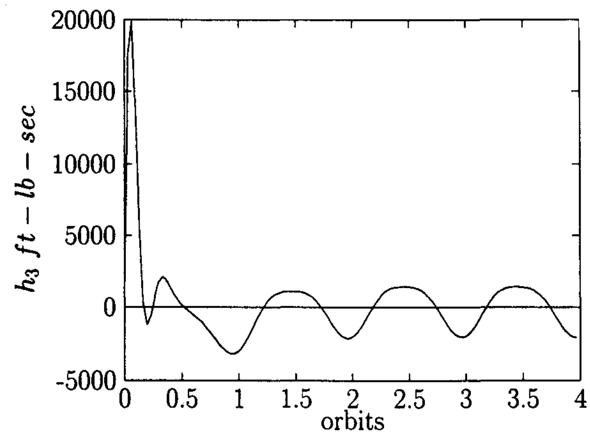

Fig. 1f Attitude regulation of nominal parameters: h_3 momentum.

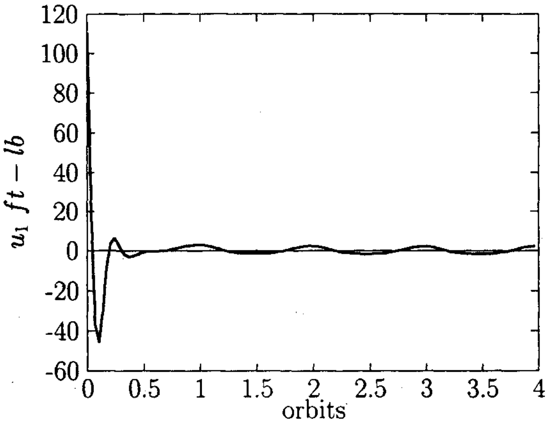

Fig. 1g Attitude regulation of nominal parameters: u_1 control.

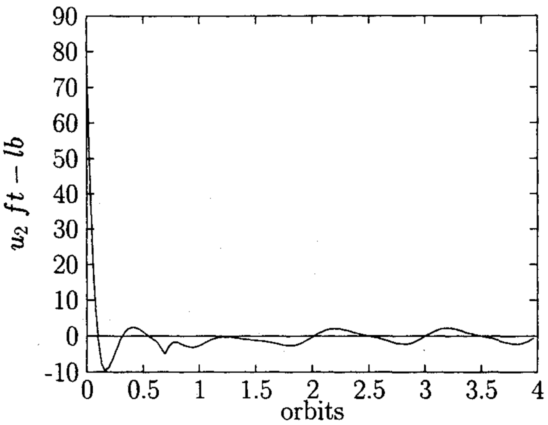

Fig. 1h Attitude regulation of nominal parameters: u_2 control.

$$x(0) = (4,\ 0.001,\ 0,\ 30,\ 0.001,\ 0,\ 3,\ 0.001,\ 0)^T$$

$$w(0) = (1,\ 0,\ -1,\ 0,\ -0.5,\ 4,\ 0,\ -2,\ 0,$$

$$-0.5,\ 1,\ 0,\ -1,\ 0,\ -0.5)^T \tag{22}$$

$$\hat{w}_i(0) = 0$$

Here, we have chosen considerable state estimation error at $t = 0$ to show the effectiveness of the observer. For the given disturbance input, using Eqs. (A3), (A7), and (A16), gives

$$(\theta_1^*,\ \theta_2^*,\ \theta_3^*) = (-0.375\ \ -7.708\ \ -1.278)\ (\text{deg})$$

$$c^T = (0.005799\ \ -0.000648\ \ -0.002832\ \ -0.000076)$$

$$d^T = (0.000648\ \ -0.005799\ 0.000038\ \ -0.001416)$$

$$a^T = (-0.675745\ \ -0.187459\ \ -0.335307\ 0.036049)$$

$$b^T = (0.187459\ \ -1.675745\ 0.018024\ \ -0.667653)$$

which can be used in Eqs. (14) and (15) to obtain analytical expressions of the asymptotic trajectory x and the control input $u = \beta(w)$. We notice that for the given functions w_{id}, both the fundamental and second harmonic components appear in the oscillatory responses of θ_1, h_2, h_3, and u. However, since the magnitudes of the first and the second elements of each of the vectors a, b, c, and d are greater than the magnitudes of the third and fourth elements, respectively, the contribution of the second harmonic terms is smaller than the fundamental component in these responses.

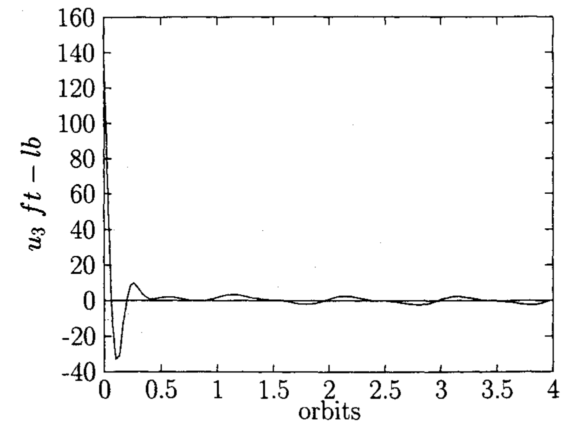

Fig. 1i Attitude regulation of nominal parameters: u_3 control.

A. Attitude Control: Nominal Parameters

Simulation was done to examine attitude regulation and momentum management capability of the control system. The initial state vector is given by Eq. (22). The complete closed-loop system (9) and (21), and the observer (18) was simulated on Cray Y-MP. For the chosen feedback gains the responses are shown in Fig. 1. We observe smooth regulation of the trajectory and indeed $x(t) \rightarrow \pi(w)$ and $u(t) \rightarrow \beta(w)$ as predicted. The state estimator error also converged to zero. The response time of the order of two orbit periods is obtained. The control magnitude and the CMG momenta are well within the specified limits.

B. Attitude Control: Off Nominal Parameters and Control Saturation

To examine the robustness of the control system to uncertainty is the inertia parameters, simulation was done for $\pm 10\%$ variation in the parameters I_i. Although oscillatory responses in the transient period were observed, the trajectories asymptotically converged to the desired values.

Simulation was also done using relatively larger perturbations in the initial states for the nominal parameters compared to the case of Sec. V.B. The initial states in this case were set to $x(0) = (8,\ 0.001,\ 0,\ 30,\ 0.001,\ 0,\ 7,\ 0.001,\ 0)^T$. Since for this larger perturbation, larger control magnitudes are expected, simulation was done by introducing saturation functions in the control channels, and thus the control input was clamped to its limiting value whenever the control magnitudes exceeded the prescribed limit. We observe that in spite of the control input saturation, attitude regulation was accomplished. Control saturation occurs only during the initial peaks of the control torque inputs. To save space the results are not shown here.

VI. Conclusions

Based on the geometric approach, the regulation of the space station was considered. For the control of the attitude angles and momenta management, an output zeroing manifold was derived. This manifold is locally invariant. In the closed-loop system this manifold was made attractive by the choice of the control law such that the system trajectories converge to the equilibrium state. An observer was designed for estimating the states of the exosystem, and the controller was synthesized using the estimated states. Numerical results were presented to show that, in the closed-loop system, large angle regulation with bounded CMG momenta can be accomplished. Although, simulation results show robustness of the controller to small parameter changes, further research related

Appendix: Proof and System Parameters

Proof of Theorem 1

In the following it will be shown that the choice of π and β according to Eqs. (14) and (15) satisfies the two conditions Eqs. (12) and (13). First note that the vector $\beta(w)$ does not contain constant terms, and $\beta(0) = 0$. Since the pitch dynamics are decoupled from the yaw and the roll dynamics, we first derive its associated output zeroing submanifold. Using Eqs. (12) and (13), we obtain the equations related to the pitch dynamics given by

$$\begin{bmatrix} \frac{\partial \pi_4(w)}{\partial w} \\ \frac{\partial \pi_5(w)}{\partial w} \\ \frac{\partial \pi_6(w)}{\partial w} \end{bmatrix} Sw = \begin{bmatrix} \pi_5 \\ f_5[\pi(w)] + b_2(-\beta_2 + w_{2d}) \\ \beta_2 \end{bmatrix} \tag{A1}$$

$$\pi_4(w) - \theta_2^*(w_{20}) = 0$$

where $\beta = (\beta_1, \beta_2, \beta_3)^T$. Since from Eq. (A1) $\pi_4 = \theta_2^*(w_{20})$ and $[\partial\pi_4(w)/\partial w]Sw = [\partial\theta_2^*(w_{20})/\partial w_{20}]0 = 0$, in view of Eq. (A1), one has $\pi_5 = 0$ and, therefore, $[\partial\pi_5(w)/\partial w]Sw = 0$. To satisfy the second equation in Eq. (A1), we choose $\beta_2 = w_{21} + w_{23}$ and

$$w_{20} = -b_2^{-1}f_5[\pi(w)] = 3n^2(I_1 - I_3)\sin\theta_2^*\cos\theta_2^* \tag{A2}$$

Solving Eq. (A2) gives

$$\pi_4(w_{20}) = \theta_2^*(w_{20}) = 0.5\sin^{-1}\{2w_{20}/[3n^2(I_1 - I_3)]\} \tag{A3}$$

We note that $\dot{\bar{w}}_i = \Lambda \bar{w}_i$. Since $\pi_6(\bar{w}_2) = n^{-1}(w_{22} + 0.5w_{24})$ from Eq. (14), one has in view of Eqs. (7) and (A1) $[\partial\pi_6(w)/\partial w]Sw = [\partial\pi_6(\bar{w}_2)/\partial\bar{w}_2]\Lambda\bar{w}_2 = w_{21} + w_{23} = \beta_2$, which establishes Eq. (A1).

Now we consider the derivation of the output zeroing submanifold associated with the yaw and roll dynamics when $\theta_2 = \theta_2^*(w_{20})$. Then Eqs. (12) and (13) give

$$\begin{bmatrix} \frac{\partial \pi_1(w)}{\partial w} \\ \frac{\partial \pi_2(w)}{\partial w} \\ \frac{\partial \pi_3(w)}{\partial w} \\ \frac{\partial \pi_7(w)}{\partial w} \\ \frac{\partial \pi_8(w)}{\partial w} \\ \frac{\partial \pi_9(w)}{\partial w} \end{bmatrix} Sw = \begin{bmatrix} \pi_2(w) \\ f_2[\pi(w)] + b_1(-\beta_1 + w_{1d}) \\ n\pi_9(w) + \beta_1 \\ \pi_8(w) \\ f_8[\pi(w)] + b_3(-\beta_3 + w_{3d}) \\ -n\pi_3(w) + \beta_3 \end{bmatrix} \tag{A4}$$

$$\pi_3(w) = 0 \qquad \pi_7(w) - \theta_3^*(w) = 0$$

From Eq. (14) we have $\pi_1 = \theta_1^* + c^T\bar{w}_1 + d^T\bar{w}_3$, and $\pi_7 = \theta_3^*(w_0)$, $\pi_8 = 0$. Thus, in view of Eq. (14),

$$\frac{\partial\pi_1(w)}{\partial w}Sw = \frac{\partial\pi_1(\bar{w}_1)}{\partial\bar{w}_1}\Lambda\bar{w}_1 + \frac{\partial\pi_1(\bar{w}_3)}{\partial\bar{w}_3}\Lambda\bar{w}_3 = c^T\Lambda\bar{w}_1 + d^T\Lambda\bar{w}_3 = \pi_2$$

$$\frac{\partial\pi_7(w)}{\partial w}Sw = \frac{\partial\pi_7(w_0)}{\partial w_0}0 = 0 = \pi_8$$

This verifies the first and fourth equation of Eqs. (A4). Now we consider the verification of the remaining equations of Eqs. (A4) using π and β from Eqs. (14) and (15).

Since $[\partial\pi_2(w)/\partial w]Sw = [\partial(c^T\Lambda\bar{w}_1 + d^T\Lambda\bar{w}_3)/\partial w]Sw = c^T\Lambda^2\bar{w}_1 + d^T\Lambda^2\bar{w}_3$ and $[\partial\pi_8(w)/\partial w]Sw = 0$, the second and the fifth equation of Eqs. (A4) give

$$c^T\Lambda^2\bar{w}_1 + d^T\Lambda^2\bar{w}_3 = (-k_1(\theta_2^*)\pi_1 + k_2\pi_8 - k_4(\theta_2^*)\pi_7 - \beta_1 + w_{1d})I_1^{-1}$$

$$0 = (-k_5(\theta_2^*)\pi_7 - k_2\pi_2 - k_3(\theta_2^*)\pi_1 - \beta_3 + w_{3d})I_3^{-1}) \tag{A5}$$

Let us choose the constant terms θ_1^* of π_1 and $\pi_7 = \theta_3^*$ such that the constant terms in Eqs. (A5) due to w_{10} and w_{30} are eliminated. Thus collecting constant terms in Eqs. (A5), gives

$$\begin{bmatrix} k_1(\theta_2^*) & k_4(\theta_2^*) \\ k_3(\theta_2^*) & k_5(\theta_2^*) \end{bmatrix} \begin{bmatrix} \theta_1^* \\ \theta_3^* \end{bmatrix} = \begin{bmatrix} w_{10} \\ w_{30} \end{bmatrix} \tag{A6}$$

It is easy to check that the solution of Eq. (A6) is

$$\begin{bmatrix} \theta_1^* \\ \theta_3^* \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} k_5(\theta_2^*)w_{10} - k_4(\theta_2^*)w_{30} \\ -k_3(\theta_2^*)w_{10} + k_1(\theta_2^*)w_{30} \end{bmatrix} \tag{A7}$$

where $\Delta = 4n^2(I_2 - I_3)(I_2 - I_1) \neq 0$. We observe that θ_1^* and θ_3^* are functions of the bias terms of the disturbance inputs. The remaining terms of Eqs. (A5), using the expressions for π_i and β_i from Eqs. (14) and (15), are

$$I_1c^T\Lambda^2\bar{w}_1 + I_1d^T\Lambda^2\bar{w}_3 = -k_1(c^T\bar{w}_1 + d^T\bar{w}_3) - a^T\bar{w}_1 - b^T\bar{w}_3 - w_{11} - w_{13}$$

$$0 = -k_2(c^T\Lambda\bar{w}_1 + d^T\Lambda\bar{w}_3) - k_3(c^T\bar{w}_1 + d^T\bar{w}_3) + n^{-1}(a^T\Lambda\bar{w}_1 + b^T\Lambda\bar{w}_3) + w_{31} + w_{33} \tag{A8}$$

Since $\bar{w}_1$ and $\bar{w}_3$ are independent functions, for equality (A8) to hold, one must set the coefficient matrices of $\bar{w}_1$ and $\bar{w}_3$ in Eq. (A8) to zero. Thus equating the coefficients of $\bar{w}_1$ and $\bar{w}_3$ in Eq. (A8), gives

$$I_1c^T\Lambda^2 + k_1c^T + a^T = [1\ 0\ 1\ 0] \tag{A9}$$

$$k_2c^T\Lambda + k_3c^T - (a^T\Lambda/n) = 0 \tag{A10}$$

$$I_1d^T\Lambda^2 + k_1d^T + b^T = 0 \tag{A11}$$

$$k_2d^T\Lambda + k_3d^T - (b^T\Lambda/n) = [1\ 0\ 1\ 0] \tag{A12}$$

Substituting a^T and b^T from Eqs. (A9) and (A11) into Eqs. (A10) and (A12), respectively, gives

$$\begin{pmatrix} c^T \\ d^T \end{pmatrix} R = \begin{bmatrix} 0 & -1 & 0 & -2 \\ 1 & 0 & 1 & 0 \end{bmatrix} \tag{A13}$$

where $R = k_2\Lambda + k_3I + (I_1\Lambda^2 + k_1I)\Lambda n^{-1}$. Here I denotes a 4×4 identity matrix. Using Λ as defined in Eq. (8), one can explicitly solve Eq. (A13) to yield

$$\begin{pmatrix} c^T \\ d^T \end{pmatrix} = \begin{bmatrix} 0 & -1 & 0 & -2 \\ 1 & 0 & 1 & 0 \end{bmatrix} R^{-1} \tag{A14}$$

Substituting Eq. (A14) in Eqs. (A9) and (A11) gives

$$\begin{pmatrix} a^T \\ b^T \end{pmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & -1 & 0 & -2 \\ 1 & 0 & 1 & 0 \end{bmatrix} R^{-1}(I_1\Lambda^2 + k_1I) \tag{A15}$$

Since $\pi_3 = 0$, the third equation of Eqs. (A4) gives $n\pi_9(w) + \beta_1 = 0$, which is satisfied using β_1 from Eq. (15), and π_9 from Eq. (14). Finally, using the last equation of Eqs. (A4), gives $[\partial\pi_9(w)/\partial w]Sw = -[n^{-1}\partial(a^T\bar{w}_1 + b^T\bar{w}_3)/\partial w]Sw = -(a^T\Lambda\bar{w}_1 + b^T\Lambda\bar{w}_3)/n = -n\pi_3 + \beta_3 = \beta_3$, in view of Eq. (15). Thus with the choice of the row vectors of a, b, c, and d according to Eqs. (A14) and (A15), Theorem 2 and hence conditions (12) and (13) of Theorem 1 are satisfied, and this establishes Theorem 2.

System Parameters, System Torque, and Disturbance Inputs

$$(I_1, I_2, I_3) = (50.28E6, 10.8E6, 58.57E6)\ \text{slug-ft}^2$$
$$(h_1, h_2, h_3) \le (20{,}000, 20{,}000, 20{,}000)\ \text{ft-lb-s}$$
$$(u_1, u_2, u_3) \le (150, 150, 150)\ \text{ft-lb}$$
$$w_{1d} = 1 + \sin(nt) + 0.5\sin(2nt)\ \text{ft-lb}$$
$$w_{2d} = 4 + 2\sin(nt) + 0.5\sin(2nt)\ \text{ft-lb}$$
$$w_{3d} = 1 + \sin(nt) + 0.5\sin(2nt)\ \text{ft-lb}$$

Parameter Vectors *a, b, c, d*

Using Eqs. (A14) and (A15), we obtain

$$c^T = (a_{11}\Delta_1 \quad -k_3\Delta_1 \quad 2\hat{a}_{11}\Delta_2 \quad -2k_3\Delta_2)$$
$$a^T = (1 + a_{11}m_1\Delta_1 \quad -k_3m_1\Delta_1 \quad 1 + 2\hat{a}_{11}m_2\Delta_2 \quad -2k_3m_2\Delta_2)$$
$$d^T = (k_3\Delta_1 \quad a_{11}\Delta_1 \quad k_3\Delta_2 \quad \hat{a}_{11}\Delta_2)$$
$$b^T = (k_3m_1\Delta_1 \quad a_{11}m_1\Delta_1 \quad k_3m_2\Delta_2 \quad \hat{a}_{11}m_2\Delta_2)$$

where $a_{11} = k_2n + k_1 - I_1n^2$, $\hat{a}_{11} = 2k_2n + 2k_1 - 8I_1n^2$, $\Delta_1 = 1/(k_3^2 + a_{11}^2)$, $\Delta_2 = 1/(k_3^2 + \hat{a}_{11}^2)$, $m_1 = I_1n^2 - k_1$, and $m_2 = 4I_1n^2 - k_1$.

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References

[1]Wie, B., Bynn, K. W., Warren, V. W., Geller, D., Long, D., and Sunkel, J., "New Approach to Attitude/Momentum Control for the Space Station," *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 5, 1989, pp. 714–722.

[2]Iyer, A., and Singh, S. N., "MFDs of Spinning Satellite and Attitude Control Using Gyrotorquers," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 25, Sept. 1989, pp. 611–620.

[3]Sunkel, J. W., and Shieh, L. S., "Optimal Momentum Management Controllers for the Space Station," *Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 6, 1990, pp. 659–668.

[4]Wie, B., Hu, A., and Singh, R., "Multibody Interaction Effects on Space Station Attitude Control and Momentum Management," *Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 6, 1990, pp. 993–999.

[5]Warren, W., Wie, B., and Geller, D., "Periodic-Disturbance Accommodating Control of the Space Station for Asymptotic Momentum Management," *Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 6, 1990, pp. 984–992.

[6]Sunkel, J. W., Sheih, L. S., and Zhang, T. L., "Digital Redesign of an Optimal Momentum Management Controller for the Space Station," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 4, 1991, pp. 712–723.

[7]Sunkel, J. W., and Shieh, L. S., "Multistage Design of an Optimal Momentum Management Controller for the Space Station," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 3, 1991, pp. 492–502.

[8]Zhao, X. M., Shieh, L. S., Sunkel, J. W., and Yuan, Z. Z., "Self-Tuning Control of Attitude and Momentum Management for the Space Station," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 1, 1992, pp. 17–27.

[9]Rhee, I., and Speyer, J. L., "Robust Momentum Management and Attitude Control System for the Space Station," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 2, 1992, pp. 342–351.

[10]Skaar, S. B., and Kraige, L. G., "Large-Angle Spacecraft Attitude Maneuvers using an Optimal Reaction Wheel Power Criterion," *Journal of the Astronautical Sciences,* Vol. 32, No. 1, 1984, pp. 47–61.

[11]Vadali, S. R., and Oh, H., "Spacecraft Station Attitude Control and Momentum Management: A Nonlinear Look," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 3, 1992, pp. 577–586.

[12]Dabbous, T. E., and Ahmed, N. U., "Nonlinear Optimal Feedback Regulation of Satellite Angular Moments," *IEEE Transactions on Aerospace and Electronic Systems*, Vol AES-18, No. 1, 1982, pp. 2–10.

[13]Singh, S. N., and de Araùjo, A. D., "Asymptotic Reproducibility in Nonlinear Systems and Attitude Control of Gyrostat," *IEEE Transactions of Aerospace and Electronic Systems,* Vol. AES-20, No. 2, 1984, pp. 94–103.

[14]Dwyer, T. A. W., and Batten, A. L., "Exact Spacecraft Detumbling and Reorientation Maneuvers with Gimbaled Thrusters and Reaction Wheels," *Journal of the Astronautical Sciences,* Vol. 33, No. 2, 1983, pp. 217–232.

[15]Singh, S. N., "Attitude Control of a Three Rotor Gyrostat in the Presence of Uncertainty," *The Journal of the Astronautical Sciences,* Vol. 35, July–Sept. 1987, pp. 329–345.

[16]Singh, S. N., and Bossart, T. C., "Exact Feedback Linearization and Control of Space Station using CMG," *IEEE Transactions on Automatic Control,* Vol. 38, No. 1, 1993, pp. 184–187.

[17]Bossart, T. C., and Singh, S. N., "Input-Output Linearization, Zero Dynamics Stability and Nonlinear Control of Space Station," *Acta Astronautica,* Vol. 29, No. 6, 1993, pp. 417–427.

[18]Isidori, A., and Byrnes, C. I., "Output Regulation of Nonlinear Systems," *IEEE Transactions on Automatic Control,* Vol. 35, No. 2, 1990, pp. 131–140.

[19]Isidori, A., *Nonlinear Control Systems: An Introduction,* Springer-Verlag, New York, 1989, pp. 347–361.